LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

SEMESTER EXAMINATION – NOVEMBER 2014

B.Sc. DEGREE EXAMINATION

MT5408 – GRAPH THEORY

Date & Time: / /2014/9.00 - 12.00 Dept. No. Max. : 100 Marks

SECTION – A

ANSWER ALL QUESTIONS: (10 x 2 = 20)

1. Show that in any graph G the number of points of odd degree is even.
2. Let G be a (p, q) graph all of whose points have degree k or $k + 1$.
3. When a $v_n - v_0$walk is said to be closed?
4. Define block.
5. Prove that every Hamiltonian Graph is $2 -$ connected.
6. Define a tree with examples.
7. Prove that every non-trivial tree G has atleast two vertices of degree 1.
8. Define eccentricity $e(v)$ and radius $r(v)$ in a connected graph G.
9. Show that $K_{3,3}$ is not planar.
10. Define $n -$ colorable graph with examples.

SECTION – B

ANSWER ANY FIVE QUESTIONS: (5 x 8 = 40)

11. (a) Let G be a k-regular bigraph with bipartition (V_1, V_2) and $k > 0$. Prove that $|V_1| = |V_2|$.
    (b) Prove that $\delta \leq \frac{2q}{p} \leq \Delta$. (4+4)
12. Let G_1 be a (p_1, q_1) graph and G_2 be a (p_2, q_2) graph. Show that
    (i) $G_1 \times G_2$ is a $(p_1 p_2, q_1 p_2 + q_2 p_1)$ graph.
    (ii) $G_1[G_2]$ is a $(p_1 p_2 , p_1 q_2 + p_2^2 q_1)$ graph.
13. Prove that a graph G is connected iff for any partition of V into subsets V_1 and V_2 there is a line joining a point of V_1 to a point of V_2.
14. Let v be a point of a connected graph G. Then show that the following statements are equivalent.
    (i) v is a cut point of G.
    (ii) there exists a partition of $V - \{v\}$ into subsets U and W such that each $u \in U$ and $w \in W$, the point v is on every $u - w$ path.
    (iii) There exists two points u and w distinct from v such that v is on every $u - w$ path.
15. Show that a line x of a connected graph G is a bridge iff x is not on any cycle of G.
16. Let G be a (p, q) graph. Then prove that the following are equivalent:
    (i) G is a tree.
    (ii) Every two points of G are joined by a unique path.
    (iii) G is connected and $p = q + 1$.
    (iv) G is acyclic and $p = q + 1$.
17. Prove that K_5 is non-planar.
18. (i) State and prove Euler's formula.
    (ii) If G is a (p, q) plane graph in which every face is an n cycle then show that $q = \frac{n(p-2)}{n-2}$.
    (5+3)

SECTION – C

ANSWER ANY TWO QUESTIONS: **(2 x 20 = 40)**

19. (a) The maximum number of lines among p points with no triangles is $\left[\frac{p^2}{2}\right]$.
    (b) Prove that $\Gamma(G) = \Gamma(\bar{G})$. (15+5)
20. (a) Prove that in any graph G, any $u - v$ walk contains a $u - v$ path.
    (b) A graph G with atleast two points is bipartite iff all of its cycles are of even length. (5+15)
21. (a) Prove that the following statements are equivalent for a connected graph G.
    (i) G is Eulerian
    (ii) Every point of G has even degree.
    (iii) The set of edges G can be partitioned into cylces.
    (b) If G is Hamiltonian, then prove that for every nonempty proper subset S of $V(G)$, $\omega(G - S) \leq |S|$, where $\omega(H)$ denoted the number of components in any graph H. (15+5)
22. (a) Prove that the following statements are equivalent for any graph G.
    i. G is 2 – colorable
    ii. G is bipartite.
    iii. Every cycle of G has even length.
    (b) Prove that every planar graph is five colorable. (8+12)

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